



# EXCERPT FROM THE PROCEEDINGS

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## OF THE TENTH ANNUAL ACQUISITION RESEARCH SYMPOSIUM SYSTEM OF SYSTEMS MANAGEMENT

### **Acquisition Management for System of Systems: Affordability Through Effective Portfolio Management**

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## Preface & Acknowledgements

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Welcome to our Tenth Annual Acquisition Research Symposium! We regret that this year it will be a “paper only” event. The double whammy of sequestration and a continuing resolution, with the attendant restrictions on travel and conferences, created too much uncertainty to properly stage the event. We will miss the dialogue with our acquisition colleagues and the opportunity for all our researchers to present their work. However, we intend to simulate the symposium as best we can, and these *Proceedings* present an opportunity for the papers to be published just as if they had been delivered. In any case, we will have a rich store of papers to draw from for next year’s event scheduled for May 14–15, 2014!

Despite these temporary setbacks, our Acquisition Research Program (ARP) here at the Naval Postgraduate School (NPS) continues at a normal pace. Since the ARP’s founding in 2003, over 1,200 original research reports have been added to the acquisition body of knowledge. We continue to add to that library, located online at [www.acquisitionresearch.net](http://www.acquisitionresearch.net), at a rate of roughly 140 reports per year. This activity has engaged researchers at over 70 universities and other institutions, greatly enhancing the diversity of thought brought to bear on the business activities of the DoD.

We generate this level of activity in three ways. First, we solicit research topics from academia and other institutions through an annual Broad Agency Announcement, sponsored by the USD(AT&L). Second, we issue an annual internal call for proposals to seek NPS faculty research supporting the interests of our program sponsors. Finally, we serve as a “broker” to market specific research topics identified by our sponsors to NPS graduate students. This three-pronged approach provides for a rich and broad diversity of scholarly rigor mixed with a good blend of practitioner experience in the field of acquisition. We are grateful to those of you who have contributed to our research program in the past and encourage your future participation.

Unfortunately, what will be missing this year is the active participation and networking that has been the hallmark of previous symposia. By purposely limiting attendance to 350 people, we encourage just that. This forum remains unique in its effort to bring scholars and practitioners together around acquisition research that is both relevant in application and rigorous in method. It provides the opportunity to interact with many top DoD acquisition officials and acquisition researchers. We encourage dialogue both in the formal panel sessions and in the many opportunities we make available at meals, breaks, and the day-ending socials. Many of our researchers use these occasions to establish new teaming arrangements for future research work. Despite the fact that we will not be gathered together to reap the above-listed benefits, the ARP will endeavor to stimulate this dialogue through various means throughout the year as we interact with our researchers and DoD officials.

Affordability remains a major focus in the DoD acquisition world and will no doubt get even more attention as the sequestration outcomes unfold. It is a central tenet of the DoD’s Better Buying Power initiatives, which continue to evolve as the DoD finds which of them work and which do not. This suggests that research with a focus on affordability will be of great interest to the DoD leadership in the year to come. Whether you’re a practitioner or scholar, we invite you to participate in that research.

We gratefully acknowledge the ongoing support and leadership of our sponsors, whose foresight and vision have assured the continuing success of the ARP:



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# System of Systems Management

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## **Acquisition Management for System of Systems: Affordability Through Effective Portfolio Management**

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# Acquisition Management for System of Systems: Affordability Through Effective Portfolio Management

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## Abstract

The lack of focus on complexity issues in System of Systems–related acquisitions prevents effective support for Better Buying Power (BBP) targets of affordability, innovation, increased productivity, and healthy competition in reducing costs and improving delivery of promised performance. The impetus is to provide the necessary analytical frameworks and associated tools that enable better informed decisions in support of BBP objectives. This paper extends our previous work in robust portfolio optimization and adopts a multi-period portfolio management approach to support the objectives of BBP. Derived from the financial engineering and operations research literature, robust multi-period portfolio management principles provide a decision-making framework that balances performance of a “portfolio” of systems, constituting, for example, a system of systems, against potential risks. The framework also balances short versus long term gains through its multi-period formulation. An illustrative example, using a Littoral Combat Ship–inspired naval warfare scenario, demonstrates application of the approach and potential use for acquisition practitioners.

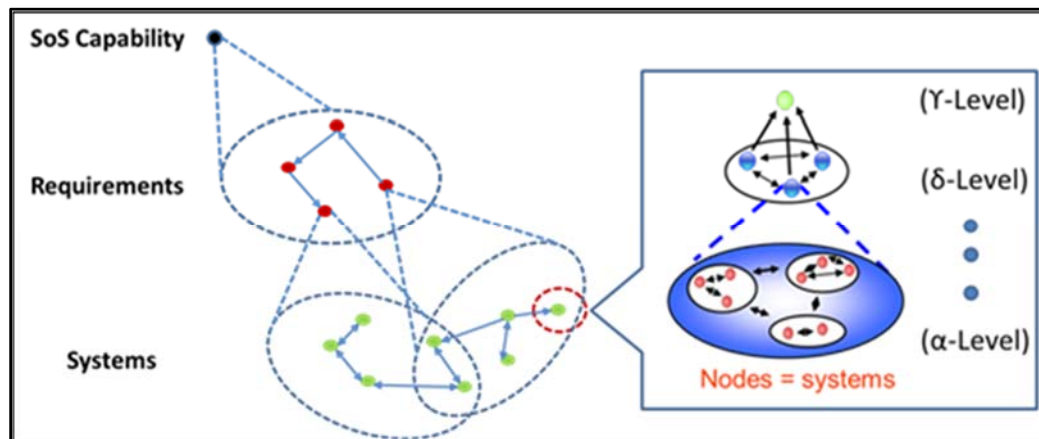
## Introduction

The U.S. Department of Defense (DoD) has emphasized a need for Better Buying Power (BBP) initiatives in tackling issues of increasing costs, schedule growth, and reduced productivity. The success of BBP policies in reducing costs have been well documented for a variety of cases that include the acquisition of Navy destroyers, reduction in production rates for the E-2D Hawkeye program, and cutting cycle times and cost of ammunitions through an improved small business acquisition strategy. However, the complexities of modern platforms that interact as a *system of systems* (SoS; Maier, 1998) present the risk of cascading modes of failure; this is due to the highly interdependent, yet operationally and managerially independent, interactions between the constituent systems. The desire to promote adequate competitions and growth of technological options in developing military capabilities has further increased the complexity of the acquisition process. This increase in complexity now includes the need to account for competitive elements in contracting, improving productivity, and reducing unnecessary redundancies. The management of Major Defense Acquisition Programs (MDAPs) through a “should cost–will cost” imperative becomes increasingly difficult as acquisition decisions must carefully balance performance and risk, and time.

The acquisition of systems with an SoS capability in mind increases the complexity. Current tools especially for this problem context are lacking. Figure 1 shows an abstraction



of the hierarchical and complex relationships among the individual layers of systems in satisfying requirements and consequently, desired overarching SoS level capabilities.



**Figure 1. System of Systems Hierarchy**

The DoD (2012) *Defense Acquisition Guidebook* (DAG) and DoD System of Systems SE guide provides fundamental guidance in tackling SoS-related acquisitions; however, these greatly lack the necessary depth and decision tools in support of BBP objectives. The lack of an effective decision support framework for managing acquisition risks has led to cascading cost overruns, schedule delays, and even program cancellations. Examples of these effects have been observed in several programs such as the Joint Strike Fighter, U.S. Army Future Combat Systems (FCS; Gilmore, 2006), and U.S. Navy Littoral Combat Ship (LCS; O'Rourke, 2011) programs. Computational decision support frameworks are needed to adequately deal with the complexity of interconnected acquisition domains and to identify optimal collections of systems that mitigate cascading risks.

### **Investment Portfolio Management: A Path to Better Buying Power**

Portfolio management techniques have been successfully applied to the management of strategic “portfolios of systems” in military acquisitions; this includes application of Real Options (RO) theory and metrics such as *Knowledge-Value Added* (KVA) that account for the value added by human and IT investments (Komorovski, Housel, Hom, & Mun, 2006). Work by Mun (2005) has developed an eight-phase process to addressing portfolio management of strategic assets. Work by Giachetti (2012) has applied stochastic techniques to managing military investments. Previous research funded by the Naval Postgraduate School (NPS) and presented at the 2012 NPS Acquisition Research Symposium (Davendralingam, Mane, & DeLaurentis, 2012), has focused on a robust portfolio management problem of maximizing a warfighter SoS portfolio performance index while preserving budgetary and compatibility constraints of underlying military assets. The robust portfolio work complements prior research efforts to include algorithmic advances, explicit consideration of data uncertainty, and inclusion of SoS architectural information within a robust investment portfolio framework. The robust portfolio methodology is adapted from financial engineering literature and leverages potential gains in overall SoS capability against cost and developmental risks in selecting “baskets” of compatible, interdependent systems.

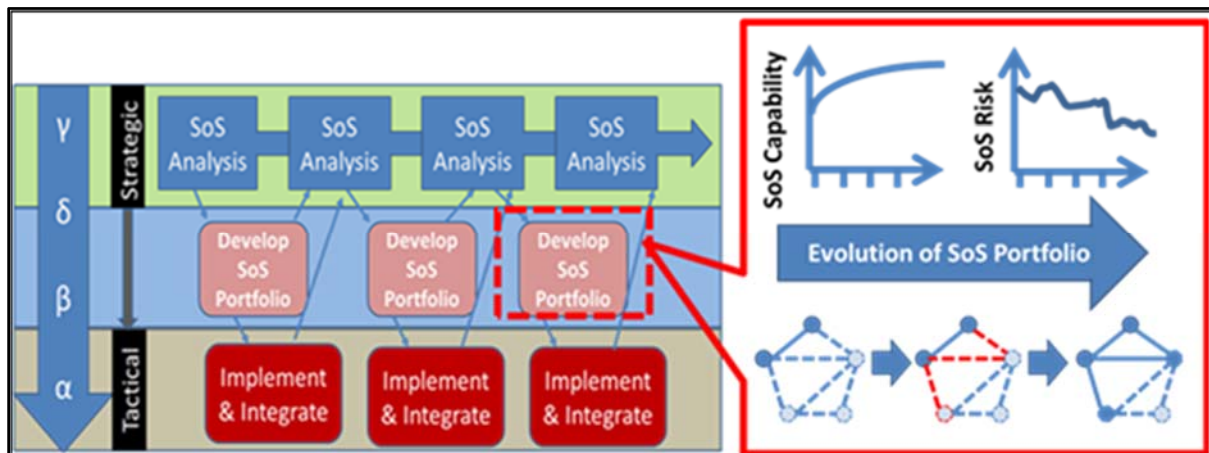
Risks and capabilities associated with system interdependencies can span the functional or physical spaces of the SoS construct and is subject to uncertainty. The developed strategy supports acquisitions, both in the pre- and post-milestone B phases, and considers current initiatives such as open architecture (OA) and competitive contracting



(e.g., fixed-price initiatives) in improving affordability and BBP objectives while considering evolving military requirements. Work in this research extends the robust portfolio approach to include a multi-period portfolio perspective. The multi-period portfolio optimization approach draws upon a rich history of algorithmic development, as noted in operations research–related literature (Powell, 2011; Bertsimas & Pachamanova, 2008; Bertsekas, 2005; Fabozzi, Kolm, Pachamanova, & Focardi, 2007; Tutuncu & Cornuejols, 2007). Its roots stem from *sequential decision-making* areas known broadly as *dynamic programming* or stochastic programming and adapts control theory methodologies to the dynamic management of resources in the interest of maximizing (or minimizing) some given metric. *Stochastic programming* focuses on issues of uncertainty whereas dynamic programming relates to the optimality of making sequential decisions; however, there has been a large degree of overlap and exchange between the two areas. Algorithmic development in these areas has been applied to a range of real-world dynamic decision-making problems that range from financial portfolio management to real-time control of vehicles.

### **A Multi-Period Decision-Making Framework**

The multi-period portfolio approach enhances the robust portfolio decision-support framework and better enables optimal acquisitions of systems in maximizing SoS-wide capabilities. The construction of an appropriate dynamic policy, in the context of an acquisition management problem, translates to identifying actions that balance the potential gains in SoS capabilities against developmental risks (e.g., cost and schedule growth risks) over a specified time horizon. Figure 2 is an abstraction of the evolution of a “portfolio of systems” that constitute an SoS, as part of the wave model (Dahmann, Rebovich, Lane, Lowry, & Baldwin, 2011).



**Figure 2. Wave Model Relation to Portfolio Evolution**

The wave model is an extension of the Department of Defense guidelines on systems engineering (SE) for an SoS that translates SoS SE core elements, interrelationships, and decision-making artifacts from a previous “Trapeze” model to a time-sequenced model representation (Dahmann et al., 2011). These architectural decisions involve the acquisition of assets in evolving the SoS capabilities to meet core objectives; the SoSE architect’s role is to explore the trade space across multiple operationally independent domains in determining suggested architectural modifications (add/remove assets) in evolving the SoS.

### ***A Multi-Period Decision Framework***

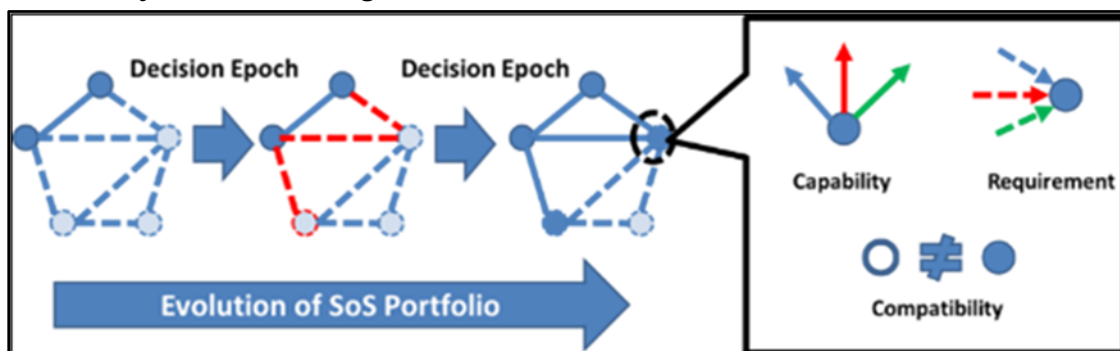
The objective of the robust multi-period portfolio framework is to allow for mathematical rigor of algorithmic techniques, transparent to the end user/practitioner, to support SoS-level acquisition decisions through identification of optimal “portfolios” of systems to be acquired in pursuit of desired SoS capabilities. While the acquisition process spans operationally and managerially independent defense groups, the tools and frameworks envisioned to support these aspects are aimed at providing adequate trade space exploration capabilities. These explorations require a domain agnostic framework, and hence intuitively resonate with the idea of treating the collection of systems across domains as a “portfolio” of systems in the SoS.

This is often the case in operations research and financial engineering applications, where underlying mathematical optimization frameworks are used to drive decision support software in assisting decision-makers (e.g., policymakers, investment specialists) in performing acquisition analysis. The concept naval warfare scenario in this paper demonstrates the application of the multi-period portfolio framework in managing the sequential acquisitions needed to propagate required capabilities while minimizing operational and developmental risks. The method illustrates the identification of optimal evolution of interconnected systems that cohesively function in providing an overarching SoS-wide capability. A robust optimization approach to the multi-period portfolio formulation addresses issues of data uncertainty.

### ***Development of a Multi-Period Investment Portfolio Model***

The acquisition (and removal) of systems in an evolving an SoS inherently involves a timeline of sequentially executed decisions. Decisions made at each epoch affect the decision options of future states, thus affecting long term performance and risks of the SoS gamut. The translation of these sequential decisions to the context of a multi-period investment model requires an adequate description of node (system) attributes; this ensures the selection of feasible portfolios that satisfy nodal requirements and minimize cascading risks. Figure 3 shows modeled generic behaviors for systems being considered in an SoS portfolio.

### ***System-of-Systems Modeling***



**Figure 3. Archetypal Node (System) Behaviors**

In Figure 3, the capabilities of an existing SoS (initial blue nodes) have the potential to evolve, based on potential connections to yet-to-be acquired systems (dashed lines and nodes). At each decision epoch, the practitioner utilizes a decision-making framework (such as the multi-period portfolio framework) to evaluate the value and risks involved in the potential acquisitions of new systems (denoted by red dashed lines). The resulting new

collection of systems that comprise the new SoS construct now includes the addition of the new systems.

An SoS is treated as a set of generic discrete nodes with the following attributes:

- *Capability (Outputs)*: Nodes have finite supply of capabilities that are limited by quantity (e.g., total power output of generator systems).
- *Requirements (Inputs)*: Nodes have individual requirements. Requirements are fulfilled by receiving capabilities from other nodes that can fulfill said set of requirements (e.g., a high powered AMDS radar requirement of energy can be fulfilled by multiple generators).
- *Compatibility*: Nodes can only connect to other nodes based on a pre-established set of rules (e.g., AMDS radar can only accept power from high capacity nuclear reactor systems on specific ships).

### **Multi-Period Investment Portfolio Formulation**

The problem statement for a multi-period investment portfolio is translated to the language of mathematical programming. The process begins with the definition of two main elements of a mathematical program, namely, the *objective function* and *constraints*. The objective function is a mathematical expression that is formulated to reflect a key performance metric of the system to be maximized (or minimized). Typical formulations of the objective function seek, for example, to minimize direct costs of operating a fleet of aircraft. For an SoS, the objective function reflects a chosen measure of performance and associated costs. The second important aspect of a mathematical program is the formulation of the constraints. The constraints reflect physical, resource, and behavioral aspects of the systems as mathematical expressions. Our initial framework for a multi-period portfolio considers a long term horizon of acquisitions with discrete decision steps that denote periods of “investment”; these investments involve the addition/removal of individual systems that comprise the overall SoS network.

The following mathematical program describes a preliminary framework for the multi-period acquisition problem:

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_{q,T}^B \right) \right) \quad (1)$$

subject to:

$$X_{q,t}^B = X_{q,t-1}^B + U_{q,t}^B + V_{q,t}^B \quad (2)$$

$$C_t^{trans} = C_q^B U_{q,t}^B + C_q^S V_{q,t}^S \quad (3)$$

$$\sum_{t=0}^T C_t^{trans} \leq \text{Budget} \quad (4)$$

$$\sum_q S_{qtC} X_{q,t}^B \geq \sum_q S_{qtR} X_{q,t}^B \text{ (Satisfy Requirements at each t)} \quad (5)$$



$$(X_{i,t}^B + \dots + X_{n,t}^B)_{j,t} = M_{j,t} \quad j = 1 \dots k \text{ (Package System Compatibility)} \quad (6)$$

$$X_{q,t}^B, X_{q,t-1}^B, U_{q,t}^B, V_{q,t}^B \in [0,1] \quad t=0 \dots T \text{ (time steps)} \quad (7)$$

where:

$w$  - weighting factor vector that weights the importance of constituent capabilities of index

$R_c$  - baseline capability level for each of the capabilities that contribute to index

$C_{q,t}^B$  - cost of acquiring system ( $q$ ) at time ( $t$ )

$C_t^S$  -cost of retiring system ( $q$ ) at time ( $t$ )

Equation 1 is the weighted objective function that seeks to maximize the end developed SoS performance index. Here, the index is related to the final state of the portfolio ( $t = T$ ) and is weighted according to the value that each capability ( $C$ ) contributes to the index (however, this can naturally reflect maximization of each stage, if necessary). The index is normalized by referencing it to some chosen reference capability set ( $R_c$ ). Equation 2 reflects the evolutionary nature of the portfolio of chosen systems ( $q$ ) at time ( $t$ ), represented by the decision vector  $X_{q,t}^B$ . Here, the decision vector is binary, to reflect discrete system choices; however, a more general setting can allow for the variables to be continuous in nature.

The terms  $U_{q,t}^B$  and  $V_{q,t}^B$  reflect decisions to “acquire” and “remove/retire” individual systems respectively, in the portfolio of systems at each decision epoch of time ( $t$ ). Equation 3 captures the “transactional” costs at each stage; this means that decisions to acquire/remove systems translate to costs associated with each that are accrued at each time step. In acquisitions, the removal cost translates to a salvage/swap cost for changing out individual systems whereas the “acquire” cost is simply the cost of purchasing and integrating a new system. Equation 4 ensures budgetary balance for total costs (transactional and acquisition) that occur.

Equation 5 ensures that the total “capabilities” from systems acquired satisfy the requirements that individual systems may have; for example, there must be adequate power generating systems selected to support selected communications systems that provide some system-wide communications capability. Conditions for Equation 5 can be enforced at each time step ( $t$ ) or at the final stage ( $t = T$ ), depending on requirements at each time step. Equation 6 enforces compatibility constraints as binary conditions for a total of ( $k$ ) set of rules; for example, the constraint that only one engine can be selected to generate power would translate to a constraint of  $x_1 + x_2 = 1$  where ( $x_1, x_2$ ) are binary variables. The rules can be applied across decision epochs, reflecting the need to have prior systems in existence, before particular upgrades can be implemented in future time steps. Equation 7 states that the decision variables are binary and that the time window consists of discrete steps from  $t = 0$  to a final time  $t = T$ . The problem formulation of Equations 1–7 constitutes a *binary integer program*, for which efficient methods of solution and commercial solvers are available.

### **Robust Multi-Period Investment Portfolio**

The multi-period formulation of Equations 1–7 are deterministic and do not consider uncertainties in the data. Real world systems are inherently driven by uncertainty and thus challenge the optimality (and feasibility) of decisions made under deterministic assumptions.



Research in mathematical programming has progressively focused more on the development of robust optimization methods to deal with manifestations of uncertainty. Robust optimization seeks to find solutions, to uncertain mathematical programming problems, that are less sensitive to parametric variations in the problem being solved. We consider uncertainties in the data for Equations 1–7, namely in the “transaction costs” of Equations 3 and 4 that reflect system addition and removal costs. We also consider uncertainties in the capabilities of each system available.

The consideration of the uncertainty in the multi-period formulation requires the use of robust optimization methods for solution. There are a range of methods that can address the uncertain linear structure of the resulting optimization problem; however, we adopt the Bertsimas–Sim (correlated case) approach for our preliminary multi-period framework. The Bertsimas–Sim method (Bertsimas & Sim, 2004) is a robust optimization approach to solving linear optimization problems with uncertain data. The method allows for a flexible adjustment in the level of conservatism of the robust solutions (termed the *Price of Robustness*) in terms of probabilistic bounds of constraint violations.

We consider the following to be a general uncertain linear program:

$$\text{maximize } c^T x \quad (8)$$

subject to:

$$A x \leq b \quad (9)$$

$$x \geq 0 \quad (10)$$

Where values  $a_{ij}$  of matrix  $A$  are uncertain and exist in the nominally symmetric bounds of  $[a_{ij} - a_{ij}, a_{ij} + a_{ij}]$ . The uncertainties are treated as *constraint-wise* uncertainties. In the correlated case, the uncertainties are modelled as the following equation:

$$\bar{a}_{ij} = a_{ij} + \sum_{k \in K_i} \bar{\eta}_{ik} g_{kj} \quad (11)$$

where  $\bar{\eta}_{ik}$  are the independent and symmetric random variables  $[-1, 1]$ , and there are  $k$  number of uncertain sources. The robust optimization problem to the correlated case can be written as the following linear optimization problem (Bertsimas & Sim, 2004):

$$\text{maximize } c^T x_j \quad (12)$$

subject to:

$$\sum_j A_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i \quad (13)$$

$$z_i + p_{ij} \geq y_j \quad (14)$$

$$-y_j \leq \sum_{j \in J_i} g_{kj} x_j \leq y_j \quad (15)$$

$$l_j \leq x_j \leq y_j \quad (16)$$



$$p_{ij}, y_{ij}, z_{ij} \geq 0 \quad (17)$$

where  $p_{ij}, y_{ij}, z_{ij}$  are the dual variables associated with the dual problem of the nonlinear formulation of the Bertsimas–Sim method (See Bertsimas and Sim [2004] for full derivation), and  $J$  is the set of uncertain coefficients. The conservatism term,  $\Gamma_i$ , is adjusted to control probabilistic guarantees of constraint ( $i$ ) violation. For example, changing  $\Gamma$ , for linear constraints that dictates power distribution flow over a network, controls the probability of net power being supplied at a prescribed level of cost. The constraint violation probability bounds for individual constraints can be approximated using the following De Moivre approximation of the binomial distribution (Bertsimas & Sim, 2004):

$$B(n, \Gamma_i) \approx 1 - \Phi\left(\frac{\Gamma_i - 1}{\sqrt{n}}\right) \quad (18)$$

where  $n$  is the  $|J_{i-}|$  and  $\Phi$  is the normal cumulative distribution function. The manipulation of  $\Gamma$  in controlling the probability of constraint violation, allows for an intuitive interpretation of the conservatism of solutions generated and permits practitioners the means of assessing solution performances against associated risk in terms of individual constraint violations.

#### **Robustification: Bertsimas–Sim (Correlated) Approach**

The robust (correlated) implementation of the Bertsimas–Sim approach in Equations 11–17 is applied to the multi-period model of Equations 1–7. The following equations described the robustified budget constraints for the multi-period model, in particular the context of budget feasibility, expressed earlier in Equation 4:

$$\underbrace{X_{q,t=T}^B + \sum_{t=0}^T C_q V_{q,t}^B}_{'c^T x_j'} + z\Gamma + \sum_{j \in J_i} P_j \leq \text{Budget} \quad (19)$$

$$z_i + p_j \geq y_j \quad (20)$$

$$-y_j \leq \sum_{j \in J_i} g_{kj} x_j \leq y_j \quad (21)$$

$$l_j \leq x_j \leq y_j \quad (22)$$

$$p_{ij}, y_{ij}, z_{ij} \geq 0 \quad (23)$$

where  $x_j$  is the concatenated decision vector  $\{X_{q,t=T}^B V_{q,t=0,1,2}^B\}$  associated with all transactions ( $t = 0, 1, 2$ ).

#### **Interpretation of Risk**

The inclusion of correlation information reflects an important contribution where protection levels of each robust constraint, in the non-correlated case assumes the simultaneous worst-case scenarios at the uncertainty bounds—a condition that is highly improbable. The correlated case accounts for the simultaneous “movements” in performance and risks across the capabilities of individual assets. Prior research has utilized a mixed integer semidefinite programming (MISDP) approach to dealing with uncertainties in the covariance matrix, a matrix that is associated with variances (risk) in system



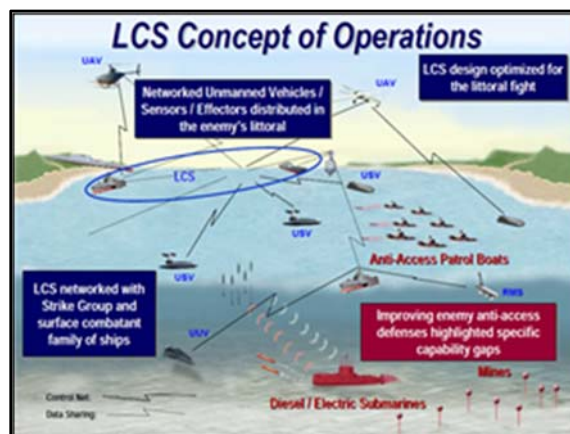


development time. However, there are very limited solvers that are able to solve MISDPs, which limits practical implementation, despite some of the computational advantages in dealing with uncertainty.

### Concept Application: Naval Acquisition Scenario

The Naval Acquisition Scenario is based on the Littoral Combat Ship (LCS) model (LCS, 2011). The LCS (Figures 4 and 5) is a naval combat vessel, developed by Lockheed Martin and General Dynamics, as a result of the Navy's dual contracting efforts to reduce cost through competition. The design of these ships seeks to provide a more agile and cost-effective solution to various near shore environment missions. These missions are executed through use of interchangeable ship packages that include Mine Warfare (MIW), Anti-Submarine Warfare (ASW), and Surface Warfare (SUW). The highly modular design of the platform allows for a great degree of operational flexibility. The modularity also translates to the ability for *open architecture* and small business initiatives to be brought to bear in reducing program costs and improving competition. Our ongoing work in this paper assumes an LCS-inspired scenario as representative "simple" SoS model where the objective is to identify potential *sequence of investment decisions* and the corresponding end collection of systems that can best maximize core capabilities of the SoS mission (in this case, MIW, ASW, and SUW).

Our highly simplified model consists of a hypothetical list of candidate systems, listed in Table 1, that are available to the Navy for acquisition. Although the numbers presented in the table are fictitious, the salient features of capability, requirements, cost, and uncertainty are nevertheless represented. Each subset of systems (listed by categories of ASW, MCM, SUW, Seaframe, Comm) represents a subset collection of systems that are available in meeting the needs of each category. The ASW, MCM, and SUW categories are the core LCS mission packages. "Seaframe" reflects the ship seaframe support options, and "Communications" represents the support communications systems available for deployment. The first five columns show capabilities of each system, and their respective numerical valuations. Columns 6 and 7 are the *Power* and *Communications* requirements needed for operation of the listed systems, in providing the capabilities listed. Also listed are the acquisition (buy) and retiring (sell/salvage) costs, along with the estimated uncertainty of each cost. We consider uncertainty in costs for this simplified problem; however, more general uncertainty in capabilities or requirements can be introduced in the same fashion.



**Figure 4. Concept of Operations**

*Note.* Image taken from presentation slides by RDML Vic Guillory of OPNAV at the Mine Warfare Association Conference (titled "Littoral Combat Ship"), May 8, 2007.



Figure 5. General Dynamics Independence Class LCS

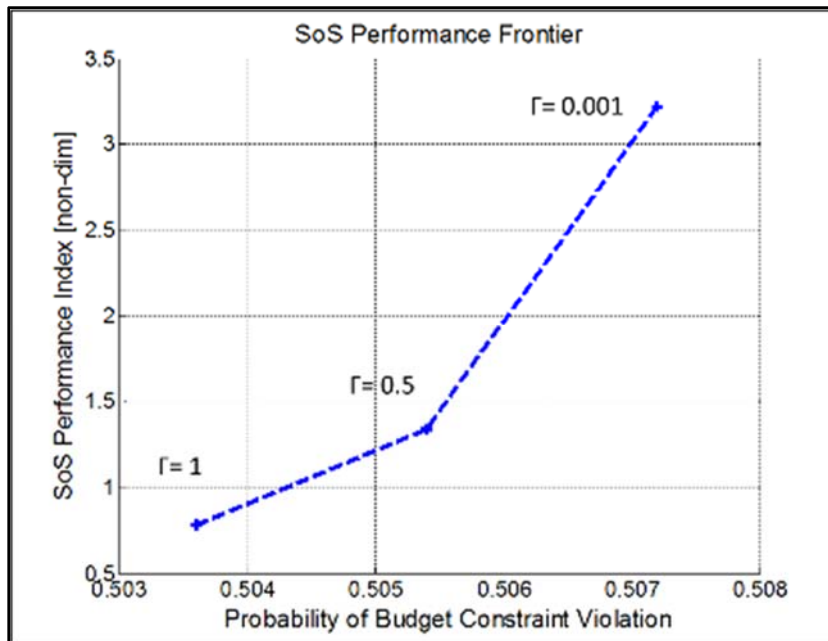
Table 1. LCS Candidate System Scenario

Category		System	Weapon Surface Anti Mine								Uncertainty Uncertainty			
			Strike	Detection	Detection	Comm	Power	Power	Comm	Acquisition	Retiring	Acquisition	Retiring	
			Range	Range	Range	Band with	Bandwith	Required	Required	Cost	Cost	Cost	Cost	
ASW	Variable Depth		0	50	0	0	0	95	100	1.00E+05	1.00E+05	9.84E+01	3.04E+01	
	Multi Fcn Tow		0	40	0	0	0	90	120	2.00E+05	2.00E+05	1.74E+02	1.83E+02	
	Lightweight tow		0	30	0	0	0	75	100	3.00E+05	3.00E+05	1.15E+02	2.37E+02	
MCM	RAMCS II		0	0	10	0	0	70	120	1.00E+05	1.00E+05	7.80E+01	9.05E+00	
	AUMDS (MH-60)		0	0	20	0	0	90	150	2.00E+05	2.00E+05	1.91E+01	1.33E+02	
	New Prototype 1		0	0	30	0	0	100	170	3.00E+05	3.00E+05	2.58E+02	1.91E+02	
SUW	N-LOS Missiles		25	0	0	0	0	0	250	1.00E+05	1.00E+05	3.49E+01	9.19E+01	
	Griffin Missiles		3	0	0	0	0	0	100	2.00E+05	2.00E+05	1.69E+02	8.05E+01	
	New Prototype 1		30	0	0	0	0	0	300	3.00E+05	3.00E+05	1.72E+02	2.91E+01	
Seaframe	Package System 1		0	0	0	0	300	0	0	1.00E+05	1.00E+05	7.02E+01	4.72E+01	
	Package System 2		0	0	0	0	450	0	0	2.00E+05	2.00E+05	1.54E+02	1.42E+02	
	Package System 3		0	0	0	0	500	0	0	3.00E+05	3.00E+05	2.41E+02	2.60E+01	
Comm.	Comm System 1		0	40	0	180	0	100	0	1.00E+05	1.00E+05	1.26E+01	3.59E+01	
	Comm System 2		0	0	0	200	0	120	0	2.00E+05	2.00E+05	1.24E+02	9.83E+01	
	Comm System 3		0	0	0	240	0	140	0	3.00E+05	3.00E+05	2.17E+02	7.00E+01	
	Comm System 4		0	0	0	300	0	160	0	4.00E+05	4.00E+05	2.20E+02	3.98E+02	
	Comm System 5		0	0	0	360	0	180	0	5.00E+05	5.00E+05	7.03E+01	4.15E+02	
	Comm System 6		0	0	0	380	0	200	0	6.00E+05	6.00E+05	4.09E+02	4.62E+02	

### Naval Acquisition Scenario: Results

The problem statement for the above LCS-inspired acquisition problem is formulated as a mathematical program that follows the robustified formulation of Equations 1–7. The resulting problem is then solved for varying values of conservatism,  $\Gamma_i$ , to reflect a range of dynamically evolving acquisitions, at each prescribed level of conservatism. Here, we assume conservatism in dealing with the costs uncertainties of acquisitions; each chosen value of  $\Gamma$  (here, three values) in this context thus reflects the probability of budget overruns occurring due to the associated costs uncertainties in each stage of acquisition. We assume a three-stage ( $t=0,1,2$ ) acquisition process, where the systems listed in Table 1 can be acquired or retired at each stage, culminating to a final “portfolio” of assets at the end of stage 3 ( $t=2$ ). Acquisition or retirement of these systems is subject to a prescribed set of rules that govern their compatibility and availabilities in time (systems only available at specific epochs) as reflected in Equation 6 of the problem formulation. Figure 6 shows the SoS performance frontier tradeoff against degree of conservatism in the budget constraint.





**Figure 6. Performance Index Frontier**

Figure 6 highlights three dynamic portfolios at conservatism level of  $\Gamma = 0.001$ ,  $0.5$ , and  $1$  respectively; increasing values of  $\Gamma$  indicate a higher degree of conservatism. Each point corresponding to a particular chosen level of conservatism reflects a sequence of acquisition decisions that lead to the final portfolio performance index denoted on the graph. The sequence of acquisitions for each level of conservatism is shown in Table 2, where “1” denotes a decision to acquire a particular system at that time step,  $t$ . Figure 7 shows the normalized capability index for each subset of capabilities that comprise the index (in this case, *weapons strike range*, *surface detection range*, and *anti-mine detection range*) of each of the optimal points in Figure 6.

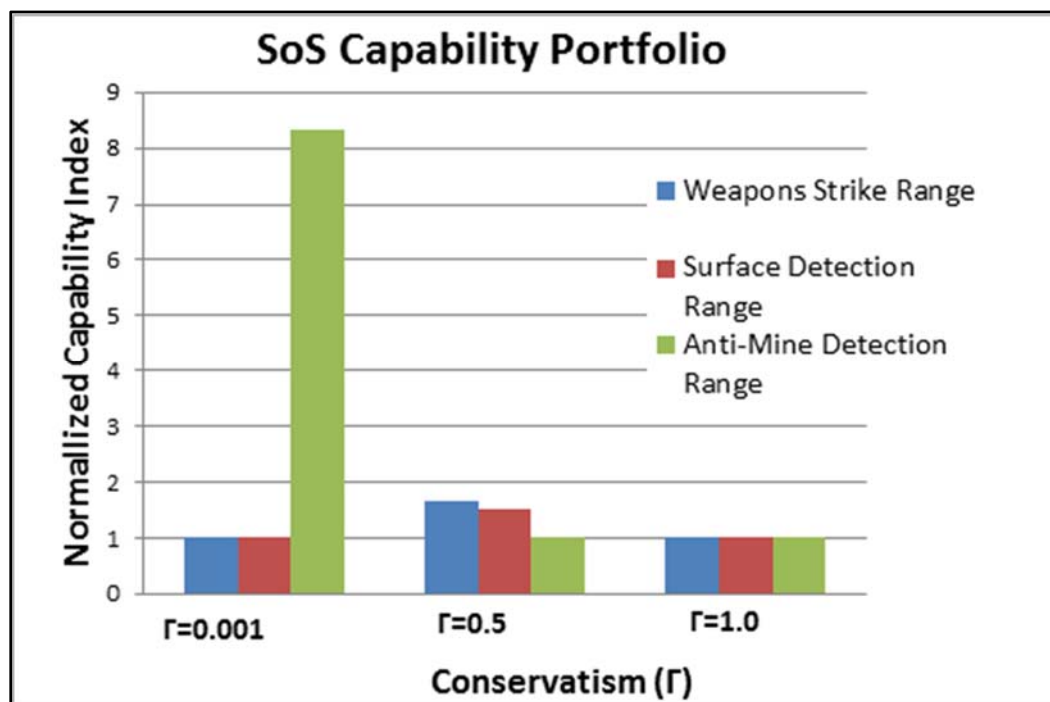
The results in Table 2 indicate evolving portfolio of systems where individual systems are acquired and retired throughout the decision epochs, preserving the satisfaction of requirements, towards maximizing the end goal of the overall SoS portfolio at time  $t = T$ . Retirements are denoted by the evolution from a previously selected state (e.g.,  $x_{jt}=1$  at  $t = 2$ ) to a state of (e.g.,  $x_{jt} = 0$  at  $t = 3$ ). At a high level of conservatism ( $\Gamma = 1.0$ ), we observe the expected case of the portfolio being constant, where the initial investments are held over the entire decision horizon without any retirement or further acquisitions; this reflects the condition where risks associated with the buy/retire transactions are deemed to be too great, hence prompting the selection of a lower capability but less financially risky acquisition strategy. At the low and mid-levels of conservatism, there is a possibility of sequential acquisitions, subject to the availability and compatibility rules between systems, that can result in higher performing portfolios but at higher prescribed level of acquisition risk.

The results of Table 2 and Figure 7 afford practitioners a candid view of the “topology” of acquisitions that can optimally be made over time, assuming a tolerance of risk for, in this case, and budgetary risk. The risk uses correlated information on the costs and is quantified as the probability associated with the budget constraint violation. The analysis result presented can be useful to decision-makers in assessing the potential dynamic purchasing/retirement decisions that need to be made in view of quantifiable uncertainties. It also allows the decision-maker to assess the trade-offs between performance and risks in decisions at each epoch of the acquisition process, while bearing independencies and

system compatibilities in mind. The mapping of the dynamic acquisition trade-space can also better inform independent acquisition groups, within an SoS, on the potential actions that various collaborative acquisition strategies can have on the overall scheme of development.

**Table 2. Portfolio Evolution at Varying Conservatism**

System Description	System Package	$\Gamma$ (Conservatism)								
		0.001			0.5			1		
		t=0	t=1	t=2	t=0	t=1	t=2	t=0	t=1	t=2
ASW	Variable Depth	0	0	0	0	0	1	0	0	0
	Multi Fcn Tow	0	0	0	0	0	0	0	0	0
	Lightweight tow	1	1	1	1	1	0	1	1	1
MCN	RAMCS II	0	0	0	1	0	0	0	0	0
	ALMDS (MH-60)	1	1	1	0	0	0	1	1	1
	New Prototype 1	0	0	0	0	1	1	0	0	0
SUW	N-LOS Missiles	0	1	1	0	0	0	0	0	0
	Griffin Missiles	1	0	0	1	1	1	1	1	1
	New Prototype 1	0	0	0	0	0	0	0	0	0
Seaframe	Package System 1	0	0	0	0	0	0	0	0	0
	Package System 2	1	1	1	1	1	1	1	1	1
	Package System 3	0	0	0	0	0	0	0	0	0
Communications	Comm System 1	1	1	1	1	1	1	1	1	1
	Comm System 2	1	0	0	1	1	1	1	1	1
	Comm System 3	0	0	0	0	0	0	0	0	0
	Comm System 4	0	0	0	0	0	0	0	0	0
	Comm System 5	0	1	1	0	0	0	0	0	0
	Comm System 6	0	0	0	0	0	0	0	0	0



**Figure 7. SoS Capability Spread at Varying Conservatism**

### Conclusions and Future Work

The development of a portfolio of systems to serve in an SoS context is a difficult endeavor. Complex interdependencies and uncertainties abound in both capabilities and

requirements of its constituent systems. There is an absence of adequate frameworks and tools to enable effective navigation of the resulting trades-spaces. Research in this paper presents a preliminary framework for a robust multi-period portfolio approach to facilitate selection of systems for acquisition in this context. The framework is naturally based on multi-period portfolio and robust optimization techniques, and it has shown promise in assessing the impact that various degrees of risk aversion (here, conservatism) on acquisition related decision epochs.

The simple LCS-inspired Naval Warfare Scenario demonstrates application of the framework; the objective is to identify optimal acquisition decisions (buy/retire) at each decision epoch, assuming various levels of conservatism in budget violations. The analysis affords practitioners a candid view of the dynamic acquisition trade-space and allows for the selection of systems at the prescribed levels of accepted conservatism. In the larger context of acquisition affordability objectives, the algorithmic framework established here has direct bearing on BBP focus areas, as listed in Table 3.

**Table 3. BBP Contributions**

<b>Better Buying Power Focus Area</b>	<b>Potential Contribution of Multi-Period Portfolio Approach</b>
Achieve Affordable Programs	<ul style="list-style-type: none"> <li>• Robust decision-making in a multi-period setting enables mitigation of risks and planning of development steps</li> </ul>
Control Lifecycle Costs	<ul style="list-style-type: none"> <li>• Robust multi-period portfolio accounts for uncertainties in transactional costs at each stage of the decision horizon.</li> </ul>
Incentivize Productivity and Innovation & Promote Effective Competition	<ul style="list-style-type: none"> <li>• Metrics such as KVA and piece-wise linear modeling of incentivizations in a multi-period setting can provide robust management of investments for non-tangible investments and incentivizations</li> <li>• Enables effective management of larger set of acquisition possibilities (e.g., contributions from SBIRs, open architectures)</li> </ul>

Our future work will leverage the robust multi-period approach by incorporating relevant metrics and perspectives, as listed in Table 3 above, to more explicitly account for sequential decision-making processes in promoting affordability in defense acquisitions.

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